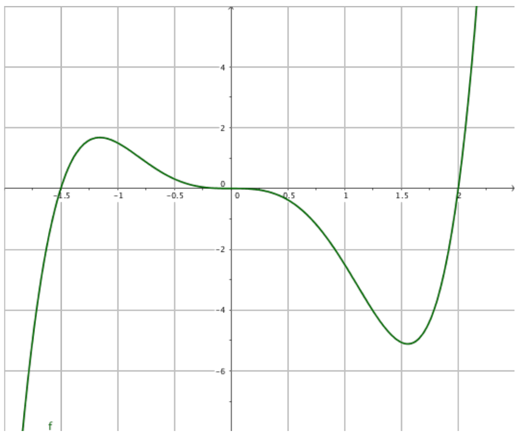
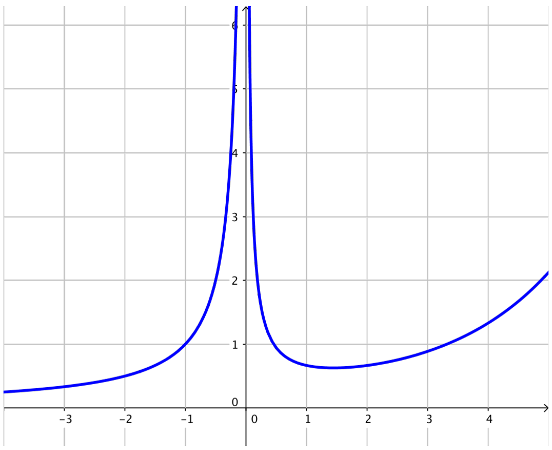
Lecture Notes: 4-3 How Derivatives Affect the Shape of a Graph (part 1)

Motivating Example: For each function graphed below, identify the regions of its *domain* where the function is increasing and where it is decreasing.



$f(x)$ is increasing:

$f(x)$ is decreasing:



$f(x)$ is increasing:

$f(x)$ is decreasing:

Question 1: Using language a middle school kid could understand, how would you explain what it means to say a function is *increasing* or *decreasing*?

Question 2: Draw a few sample tangent lines to each graph above. What is the relationship between the slope of the tangent lines and whether the graph is increasing or decreasing?

Increasing/Decreasing Test

(a) If ________________ on an interval, then the function $f(x)$ is **increasing** on this interval.

(b) If ________________ on an interval, then the function $f(x)$ is **decreasing** on this interval.

PRACTICE PROBLEM 1: Let $g(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

1. Use the Increasing/Decreasing Test to find the intervals where $g(x)$ is increasing and decreasing.

2. Sketch the graph on your calculator to check that your answer above is correct.

3. What do you observe about the relationship between *local maximums, local minimums* and intervals of *increase* and *decrease*? Make an explicit conjecture.

4. Go back and look at the examples at the top of page 1 and see if you need to amend your conjecture above.

QUESTION 3: What is a critical number again?

The First Derivative Test: Suppose that c is a critical number of a continuous function $f(x)$.

a) If ____ at c, then f has a **local maximum** at c.

b) If ____ at c, then f has a **local minimum** at c.

c) If ____ at c, then f has no local maximum or minimum at c.

Using the work from the previous PRACTICE PROBLEM 1, fill in the blanks below for $g(x) = 3x^4 - 4x^3 - 12x^2 + 5$.

(a) ________________ is a local minimum of $g(x)$ that occurs at ________________

(b) ________________ is a local minimum of $g(x)$ that occurs at ________________

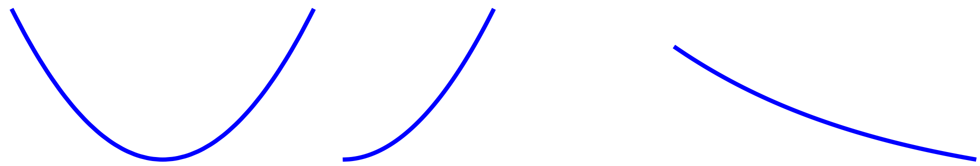
(c) ________________ is a local maximum of $g(x)$ that occurs at ________________

PRACTICE PROBLEM 2: Sketch the graph of a function $h(x)$ satisfying all the properties below:

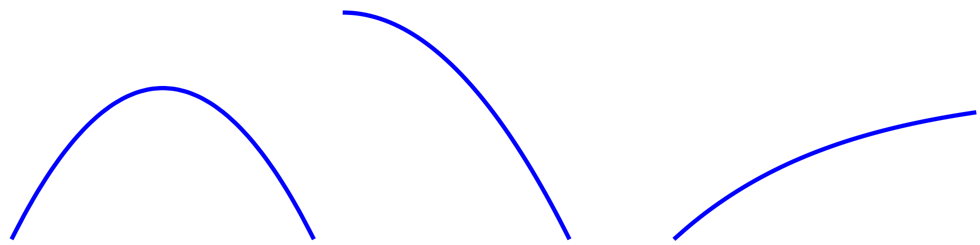
1. domain $(-\infty, \infty)$
2. $h'(x) > 0$ on $(-\infty, 0) \cup (2, \infty)$
3. $h'(x) < 0$ on $(0, 2)$
4. $h'(0)$ is undefined, $h'(2) = 0$

MOTIVATING EXAMPLES: On the sample graphs below, sketch some rough tangent lines. Sketch multiple tangents on each graph. Make rough approximations of the slopes of these tangents.

concave up
pictures



concavedown
pictures



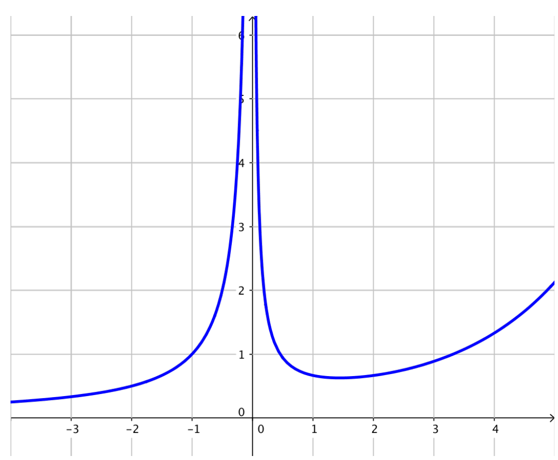
QUESTION 4: How does the relationship between the tangent line and the graph to which it is tangent differ depending on whether the graph is concave up or concave down?

QUESTION 5: If the graphs above are of a function, say $f(x)$, what can you say about its derivative $f'(x)$?

QUESTION 6: Estimate the intervals where each function below is concave up and concave down:

$f(x)$ is concave up:

$f(x)$ is concave down:



$f(x)$ is concave up:

$f(x)$ is concave down:

DEFINITION: A point P on a curve $y = f(x)$ is called an **inflection point** if f is continuous there and the curve changes from concave upward to concave downward or vice versa at P.

QUESTION 6: Do the graphs on the previous page have any inflection points?

CONCAVITY TEST & INFLECTION POINTS: Let $f(x)$ be a function defined on an interval I.

a) If ____________________ (that is: ___________________________) for all x in I, then the graph of f is concave upward on I.

b) If ____________________ (that is: ___________________________) for all x in I, then the graph of f is concave downward on I.

PRACTICE PROBLEM 3: Let $f(x) = 2x^3 - 3x^2 - 12x$. Find the intervals of concavity and the inflection points.